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ABSTRACT

This study compared the acquisition of an initial place value skill when presented in a concrete, semiconcrete, abstract teaching sequence to acquisition of the same skill when presented at the abstract level only. The 24 subjects were elementary and middle school students (ages 8-13) with learning disabilities who were randomly assigned to experimental and control groups. A 2x3 mixed design with one between (treatment) and one within (performance over time) group factor was used. A significant main effect for the treatment was found on the acquisition measure, suggesting that the concrete to abstract teaching sequence is more effective than abstract teaching alone for this concept with this population. The use of manipulatives and pictorial representations positively affected skill acquisition, maintenance, and retention, thus supporting at least as much instructional emphasis on concept teaching as on computation skills. No significant difference was found on the generalization measure. Appended are a sample posttest and a sample teaching script for a concrete lesson. (Contains 23 references.) (DB)

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Comparing the Concrete to Abstract Teaching Sequence to Abstract Instruction for Initial Place Value Skills [1, 2]

Susan K. Peterson, Program Manager
Cecil D. Mercer, Co-director
Jennifer Tragesh, Liaison Consultant
Lawrence O'Shea, Associate Professor

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UNIVERSITY OF FLORIDA

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December, 1987

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THE UNIVERSITY OF FLORIDA

MULTIDISCIPLINARY DIAGNOSTIC AND TRAINING PROGRAM (MDTP)

The MDTP is administered through a joint effort by Shands Teaching Hospital and the Department of Special Education at the University of Florida. The MDTP staff is composed of professionals from the fields of pediatric neurology, education, school psychology, and speech and language pathology. The MDTP has specified elementary school students with diverse medical, learning, and/or behavorial problems as its primary population. Major responsibilities of the MDTP are to use all appropriate disciplines to provide diagnostic and intervention services to school systems referring students, train education and health professionals at the preservice and inservice level, and assist parents of students experiencing difficulty in school.

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Preface

The purpose of this study was to compare the acquisition of an initial place value skill when presented in a concrete, semiconcrete, abstract teaching sequence to acquisition of the same skill when presented in the abstract level only. Measures of skill acquisition, maintenance, retention and generalization were included. The subjects included in the study were learning disabled elementary and middle school students.

Students were randomly assigned to the experimental and control groups. A 2X3 mixed design with one between (treatment) and one within (performance over time) group factor was used. A multivariate analysis of variance was computed to determine whether differences existed among the levels of the experimental treatments. Due to obtained significance ($p<.05$), two follow-up univariate analyses of variance were computed. A significant main effect for the treatment variable was found on the acquisition measure. No significant difference was found on the generalization measure. Students who were taught place value using the concrete to abstract teaching sequence performed significantly better on three posttest measures than students who were taught the same skill abstractly.

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Teaching Learning Disabled Students Place Value Using The Concrete to Abstract Sequence

Traditionally, the reading and language problems of learning disabled students have received extensive attention whereas mathematical problems have not been stressed (Bartel, 1982). This lack of emphasis is somewhat surprising since the 1977 Federal Register includes mathematics calculation and mathematics reasoning as primary disability areas used to identify students with learning disabilities (USOE, 1977).

In a survey of special education teachers, Carpenter (1985) found that learning disabled (LD) resource teachers spend approximately one-third of their instructional time teaching mathematics. In addition, Carpenter (1985) reported that the majority of LD teachers felt they were not competent in basic skills needed for teaching math to LD students. It is apparent that the mathematical problems of LD students are widespread, and systematic investigations in this area are needed.

Several educators (Cawley, Fitzmaurice, Goodstein, Lepore, Sedlak, & Althaus, 1976; Jones, Thornton, & Toohey, 1985; Thornton & Toohey, 1985) have examined the mathematical deficits of learning disabled students and have suggested numerous methods for teaching these students. Implicit in these works is an emphasis on teaching LD students to understand the concepts of mathematics prior to the memorization of facts, algorithms, and operations. In essence, the teaching principle that "understanding precedes memorization" is stressed.

Much of the mathematics literature supports the position that math instruction should focus on helping students understand concepts. To accomplish this, many math experts believe that math instruction should feature a concrete (use of manipulatives) - semiconcrete (use of pictorial representations) - abstract (use of numbers only) teaching sequence. Burton (1984) and Suydam and Higgins (1977) report the idea that mathematical learning progresses through a concrete to abstract sequence of understanding is reported in writings as early as the nineteenth century. Pestalozzi suggested that children must experiment in the concrete before applying abstract rules or exercises (Cajori, 1896). Bruner (1966) and Piaget and Inhelder (1958) identified the sequence as enactive, iconic, and symbolic. More recently, Underhill, Uprichard, and Heddens (1980) and Reisman (1982) refer to the sequence components as concrete, semiconcrete, and abstract. To date, the position that math instruction should feature a concrete - semiconcrete - abstract teaching sequence is widely purported.

Although mathematicians promote this teaching sequence, very little empirical data have been gathered to validate its use. The existing studies typically compare concrete instruction to either semiconcrete or abstract instruction rather than examining the effectiveness of the teaching sequence (Armstrong, 1972; Fennema, 1972; Prigge, 1978; Scott & Neufeld, 1976; Smith, Szabo, & Trueblood, 1980). Moreover, research involving the use of this teaching sequence with learning disabled students is nonexistent. The increased emphasis on math instruction for learning

disabled students (Carpenter, 1985) clearly demonstrates the need for more research in this area.

The purpose of the present study was to investigate the effectiveness of teaching learning disabled students place value through a conceptual sequence which includes the three levels of understanding (i.e., concrete, semiconcrete, and abstract) compared to the effectiveness of teaching learning disabled students the same skill at the abstract level without manipulatives or pictorial representations. The effect of these two procedures on initial skill acquisition, maintenance, retention, and generalization to a higher skill was measured.

Method

The study was divided into three phases. Phase one was a training period for the teachers involved in the study. Phase two involved nine days of direct instruction. Direct instruction methodology was used to maximize academic performance. The final phase consisted of post-treatment skill acquisition, maintenance, retention, and generalization measurement.

Subjects

The subjects for this study were learning disabled elementary and middle school students. There were 20 males and 4 females ranging in age from 8 to 13. All subjects received mathematical instruction in special education classrooms located in Florida. Of the 24 subjects included, 19 were in self-contained classrooms for the learning disabled, 4 were

temporarily placed in a self-contained diagnostic classroom, and 1 was in a resource room setting.

Additional subject charcteristics are summarized in Table 1 per the recommendation of Smith et al. (1984). These authors suggest including sex, age, race, socioeconomic status (SES), intelligence quotient (IQ) scores, and achievement scores when describing learning disabled subjects.

Table 1

Summary of Subject Characteristics

Experimental	Control	Experimental	Control
Numbers:	Numbers:	I.Q.:	I.Q.:
male 10	male 10	mean 89.0	mean 91.5
female 2	female 2	range 73-103	range 80-119
total 12	total 12	test(s) WISC-R	test(s) WISC-R
		used K-ABC	used K-ABC
Age:	Age:	Math achievement:	Math achievement:
mean 10.5	mean 10.3	mean 74.45	mean 70.45
range 9-13	range 8-12	range 64-88	range 60-83
		test(s) WRAT-R	tests(s) WRAT-R
		used K-TEA	
Race:	Race:	Reading achievement:	Reading achievement:
Anglo 9	Anglo 7	mean 67.27	mean 65.36
Hisp. 0	Hisp. 0	range 48-81	range 55-92
Black 3	Black 5	test(s) WRAT-R	test(s) WRAT-R
		used K-TEA	used
SES:	SES:	Spelling achievement:	Spelling achievement:
high 4	high 2	mean 63.36	mean 69.91
middle 1	middle 0	range 51-78	range 54-94
low 7	low 10	test(s) WRAT-R	test(s) WRAT-R
		used K-TEA	used

Note: Achievement reported in standard scores.

All subjects participated in a two-part screening procedure to determine eligibility for the study. Screening occurred to ensure the students participating did not have preexisting knowledge regarding the acquisition and generalization of place value skills. To be included students had to score 70% or lower on both sections (i.e., acquisition and generalization) of a teacher-made place value measure. Subjects who qualified for inclusion in the study were randomly assigned to the treatment and control groups.

Instrumentation

In this study, three teacher-made research instruments were used. Each instrument contains two parts: one to measure acquisition and one to measure generalization (see Appendix A). The instruments were administered immediately following instruction, 1 week later to measure maintenance, and 3 weeks after instruction to check retention.

Materials

Materials used with the learning disabled students receiving concrete instruction included plastic unifix cubes produced by Philograph Publications LTD (1974), place value sticks (i.e., popsicle sticks), and teacher-made place value strips. Materials used for semiconcrete instruction were worksheets from Invitation to Mathematics (Scott, Foresman and Company, 1985) and from Macmillan Mathematics (Macmillan Publishing Company, 1985). The worksheets selected provided various pictorial representations of place value problems (e.g., bundles of sticks). Materials used for abstract instruction included Invitation to Mathematics, Macmillan Mathematics, and teacher-made worksheets

without any pictorial representations (i.e., worksheets only presented numbers).

Procedure

Prior to the instructional implementation phase the five teachers involved in this research attended several training workshops. The participants were introduced to the direct instruction model, research regarding effective teaching, and the concrete to abstract teaching sequence. Moreover, they were trained to implement the provided instructional activities. Teachers had to demonstrate mastery of the instructional procedures before beginning the actual treatment with study participants. An interval recording system was used by two independent observers to measure each teacher's ability to follow the teaching script, follow the sequence, pace fluently, and use materials correctly. Interobserver agreement was 99%.

Instruction. The learning disabled students in both the experimental and control groups were taught to identify how many ones and/or tens are in a double digit number using a four-step direct instruction model. The steps were

1. Provide an advance organizer (i.e., previous days work is mentioned, upcoming lesson is described, and materials to be used are displayed).
2. Demonstrate and model the skill (i.e., student observes the task being performed).
3. Provide guided practice (i.e., student practices the task with immediate teacher feedback available).

4. Provide independent practice (i.e., student practices task without teacher assistance).

Each of the nine lessons for both groups was scripted and lasted between ten and fifteen minutes. One lesson per day was presented. Progression from one lesson to the next was dependent upon the student achieving a predetermined criterion for skill performance (i.e., at least 80% accuracy) on individual lesson objectives leading to the terminal objective (i.e., identifying how many ones and/or tens in a double digit number). Each teacher involved in the study taught students from both the control and experimental groups. This coupled with the use of teaching scripts controlled for teacher effect (see Appendices B-D).

The only difference between the experimental and control group treatment was the experimental group received three lessons using concrete manipulatives, three lessons using semiconcrete instruction and then three lessons with abstract level instruction. The control group received nine lessons at the abstract level.

Posttreatment. After the instructional implementation was completed, a posttest was administered to each subject in both groups to measure skill acquisition. Additionally, each student's ability to generalize newly learned information to a different stimulus (i.e., three- and four-digit numbers) was tested. The following week skill acquisition and generalization were again measured for maintenance. Three weeks after instruction ceased, retention and generalization were again tested.

Design

The experimental design used in this study is a 2 X 3 mixed design with one between (treatment) and one within (performance over time) group factor (Meyers, 1979). The multivariate analysis of variance (MANOVA) procedure was used to test the effects of the independent variables on both dependent variables (i.e., place value acquisition and place value generalization) simultaneously taking into consideration the correlation between the two (see Table 2).

Table 2

Summary of Multivariate Analysis of Variance for One- and Three Level Instruction

Source	Hotellings T	F	df	p
Between Subjects				
A	9.41713	4.49	2,21	.0237*
Within Subjects				
B	11.3374	2.45	4,19	.0816
Interaction				
AXB	4.76209	1.03	4,19	.4182

*Significant at the $p < .05$ level.

Results

The MANOVA yielded a significant main effect for instructional treatment [$F(2, 21)=4.49$; $p=.0237$]. Therefore two follow-up univariate analyses of variance were calculated; one for the acquisition and one for the generalization measure (see Tables 3 and 4).

A significant main effect for the instructional treatment variable was found on the acquisition measures [$F(1, 22)=8.79$; $p=.0072$]. Students who received the concrete to abstract instructional sequence identified ones and tens in double digit numbers significantly better than the students ones who received abstract level instruction.

Table 3

Summary Table for the Univariate Analysis of Variance for Acquisition

Source	df	SS	F	p
Between Subjects				
A	1	138.88	8.79	.0072*
Error	22	347.77		
Within Subjects				
B	2	25.75	.98	.3845
AXB	2	23.69	.90	.4143
Error	44	579.88		

*Significant at the $p < .05$ level.

Table 4

Summary Table for the Univariate Analysis of Variance for Generalization

Source	df	SS	F	p
Between Subjects				
A	1	174.22	1.66	.2111
Error	22	2310.22		
Within Subjects				
B	2	58.02	3.45	.0406*
AXB	2	3.69	.22	.8038
Error	44	370.27		

*Significant at the $p < .05$ level.

No main effect difference for the instructional treatment variable was found on the generalization measure [$F(1, 22)=1.66$; $p=.2111$]. Students in the experimental and control groups were statistically similar when performing the untaught skill of identifying ones and tens in three- and four-digit numbers. Means and standard deviations for levels of treatment and performance over time are presented in Table 5.

Related Findings

Examination of the raw scores for generalization suggests that the experimental group generalized with greater proficiency than the control students. Gain scores for generalization were calculated to reflect growth between the screening and posttest measures for the experimental and control groups. They were 47.08% and 26.25%, respectively. The gain scores reflecting change between the screening and maintenance devices

Table 5

Means and Standard Deviations for Each Dependent Measure by Experimental Condition

Instructional Treatment	Dependent Variables	Performance Over Time		
		Posttest	Maintenance	Retention
One-Level Instruction	Acquisition	16.08 (7.329)	18.75 (1.712)	16.5 (5.214)
	Generalization	6.167 (4.629)	8.5 (6.186)	8.333 (6.651)
Three-Level Instruction	Acquisition	19.83 (.5774)	19.92 (.2887)	19.92 (.2887)
	Generalization	9.75 (8.081)	11.0 (6.382)	11.58 (5.807)

were 53.33% for the experimental group and 37.92% for the control group. The gain scores between the screening and retention generalization measures were 56.25% for the experimental group and 37.08% for the control group. Thus, the experimental group performed 20.83%, 15.41%, and 19.17% better than the control group on the generalization task (see Table 6).

Table 6

Mean Generalization Gain Scores

	Control Group	Experimental Group
Screening to Posttest	26.25% (5.25 problems)	47.08% (9.42 problems)
Screening to Maintenance	37.92% (7.58 problems)	53.33% (10.67 problems)
Screening to Retention	37.08% (7.42 problems)	56.25% (11.25 problems)

The within-group variability prevented this difference from being statistically significant. Although some students did generalize with 100% accuracy; others did not generalize at all. Those who failed to generalize substantiate earlier research (Stokes & Baer, 1977) which suggests that special education students must be taught to generalize.

The difference in gain scores does, however, denote a trend toward spontaneous generalization among the experimental group students. Thus, instructionally it makes sense to use the concrete to abstract sequence when generalization is part of the teaching objective. Teachers must, however, be prepared to incorporate additional generalization practice for some students.

Discussion

The results of this study suggest that the concrete to abstract teaching sequence is more effective than abstract teaching with learning disabled students who are acquiring initial place value skills. The use of manipulatives and pictorial representations positively affects skill acquisition, maintenance, and retention. These findings are particularly relevant when considering the current educational emphasis on basic skills, minimum competency testing and progression through basal math programs. Unfortunately, teachers frequently feel compelled to cover prespecified amounts of material at a rapid pace. Correct computational answers are the primary measure for student success in mathematics and are frequently used to determine readiness for teaching a new skill. Concept teaching should receive as much emphasis as computation with learning disabled students who struggle to acquire and retain place value skills. Further research is needed to validate the concrete to abstract teaching sequence with other math skills and to examine in greater detail the effect this sequence has on generalization skills of learning disabled students.

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Appendix A

Teacher-Made Posttest Sample

Acquisition

1.	23	How many ones are in this number?	_____
2.	43	How many ones are in this number?	_____
3.	16	How many tens are in this number?	_____
4.	58	How many ones are in this number?	_____
5.	33	How many ones are in this number?	_____
6.	47	How many ones are in this number?	_____
7.	85	How many tens are in this number?	_____
8.	74	How many tens are in this number?	_____
9.	10	How many ones are in this number?	_____
10.	73	How many tens are in this number?	_____
11.	50	How many ones are in this number?	_____
12.	82	How many ones are in this number?	_____
13.	51	How many ones are in this number?	_____
14.	91	How many tens are in this number?	_____
15.	37	How many tens are in this number?	_____
16.	48	How many ones are in this number?	_____
17.	85	How many tens are in this number?	_____
18.	40	How many ones are in this number?	_____
19.	37	How many ones are in this number?	_____
20.	29	How many tens are in this number?	_____

Generalization

1.	454	How many ones are in this number?	_____
2.	4755	How many tens are in this number?	_____
3.	154	How many tens are in this number?	_____
4.	568	How many ones are in this number?	_____
5.	7867	How many tens are in this number?	_____
6.	8637	How many tens are in this number?	_____
7.	438	How many ones are in this number?	_____
8.	871	How many tens are in this number?	_____
9.	986	How many ones are in this number?	_____
10.	2345	How many ones are in this number?	_____
11.	864	How many ones are in this number?	_____
12.	561	How many tens are in this number?	_____
13.	6743	How many tens are in this number?	_____
14.	763	How many ones are in this number?	_____
15.	3456	How many ones are in this number?	_____
16.	6873	How many tens are in this number?	_____
17.	154	How many ones are in this number?	_____
18.	3910	How many tens are in this number?	_____
19.	5467	How many tens are in this number?	_____
20.	875	How many ones are in this number?	_____

Appendix B

Sample Teaching Script for Concrete Lesson

Lesson One: Identifying Ones and Tens Concretely with Cubes

Materials: plastic cubes, place value cards, vis-a-vis pen, probe, chart

Advance Organizer

"Today we're going to practice counting ones and tens using cubes. This will help us understand the meaning of two-digit numbers such as 15, 24, 32, 67." (Write these sample numbers on blackboard as they're said.) **"Notice these numbers have two digits. One digit is the ones** (point to the ones) **and the other is the tens** (point to the tens). **Before we start let me show you the materials we'll use. These are cubes. What are they?. . . Yes, cubes. These are place value cards. What are they? . . . Yes, place value cards. Good! Let's begin."**

Demonstrate/Model

(Have three groups of cubes on the table; one group of 23, one of 15, and one of 32.) **"I'm going to count the first group of cubes and put them in groups of tens and ones."** (Count 23 cubes and then group them in tens and ones. Interlock the cubes that represent tens. Leave the ones cubes unconnected.) **"Now I'm going to write how many tens I counted. How many tens? . . . Yes, I counted two tens."** (Write the 2 on place value chart.) **"How many ones do I have? . . . Yes, I**

counted three ones." (Write 3 on the place value chart.) **"Another way to write this number is 23."** (Write 23 on blackboard.) **"This number is twenty-three. The number on the far right is in the ones place. The number two places over** (count and point) **is in the tens place. What number is two places over? Yes, the tens."** (Repeat this procedure demonstrating 15 and 32.)

Guided Practice

(Give the student 35 cubes.) **"You count and put the cubes in groups of tens and ones."** (Teacher and student each count 35 cubes and group them by tens and ones.) **"How many tens did we count? . . . We counted three tens. Write 3 in the tens column on your place value chart . . . . How many ones did we count? We counted five ones. Write 5 in the ones column . . . Another way to write the 3 tens and 5 ones is 35. Write 35 on the other side of your card. Where is the ones place? Yes, on the far right. Where is the tens place? Yes, two places over."** (Repeat this procedure with the numbers 43, 19, and 20.)

Independent Practice

(Give the student 26 cubes.) **"Now you count and put the cubes in groups of tens and ones. Remember to record the number on your place value card."** (Have students repeat independent practice with the numbers 72, 10, and 12.)

Criterion

Four out of four or 100% performed correctly. Repeat practice until criterion is met.

Administer 1-minute probe. Chart performance.

Appendix C

Sample Teaching Script for Semiconcrete Lesson

Lesson Four: Identifying Tens and Ones Semiconcretely With Stick Pictures

Materials: four worksheets, probe, chart

Part One

Advance Organizer

"I'm very proud of the work you've been doing with place value. You can show me tens and ones using plastic cubes (hold up cube), **place value sticks** (hold up stick), **and place value strips** (hold up strip). **That's very good. Today we're going to practice counting pictures of place value sticks. We'll be looking for tens and ones. We'll complete four worksheets together and then take our 1-minute timing."** (Distribute worksheet number one.) **"Put your finger on the bear . . . Good following directions. This bear's name is Super 10. He's named Super 10 because he loves bundles of tens. Look at the flag he's carrying. How many bundles of tens are on the flag? . . . Yes, there are two. So how many tens are there? . . . Yes, there are two. So how many tens are there? . . . Yes, two. And how many ones are there? . . . That's right, zero. So two tens and zero ones is written like this."** (Point to the number 20.) **"Two tens and zero ones is another name for? . . . Yes, 20. Very good. Now look at my sheet."** (Point

to one bundle of ten.) **"How many tens? Yes, one. How many ones left over? . . . Right, there are no ones. So one bundle of ten and zero ones is another name for?. . . Yes, ten."** (Point to the number ten. Repeat this oral practice for the numbers 20 through 90.) **"You did a super job. Now we're ready to begin our worksheet."**

Demonstrate/Model

"Look at my sheet. I'm going to count the number of tens." (Point as you count.) **"One, two, three, four. So, I write four next to the word tens like this."** (Trace the four.) **"There are four tens. Since there are no ones to count, I know the number is** (trace the 40) **four tens and zero ones or 40. So, first I counted and wrote the number of tens. Then I wrote the other name which includes tens and ones."**

Guided Practice

"Now you count the bundles of ten. How many?. . . Yes, four. Trace the four next to the word tens. So what is the other name? . . . Yes, four. Trace the four next to the word tens. So what is the name?. . . Yes, forty. Trace the four tens and zero ones. Good job. Let's do the next one together. How many tens? . . . Yes, two. Write two next to the word tens. The other name for two tens is two tens and zero ones or 20. Write this other name in the blank . . . Good work! Now I'd like you to finish this

page and worksheet number two by yourself. When you finish put your pencil down, so I'll know you're ready to go on."

Independent Practice

(Distribute worksheet number two.) **"The problems on this sheet are done the same way. Remember to do your best work."**

Part Two

Advance Organizer

"You did a good job working by yourself. Now we're ready to go on to worksheet number three." (Distribute worksheet three.) **"We're going to practice counting tens and ones. Put your finger on the first bear picture . . . Good following directions. Super 10 bear has a lot of sticks. He's counting groups of tens. So far he has two tens. Now look at the next picture. We see he counted three groups of ten and didn't have enough sticks to make another ten. So, how many tens?. . . Yes, three. And how many ones left over? . . . Yes, five."** (Point to numbers.) **"So he wrote three under the tens and five under the ones. Where is the ones place in this number? . . . Yes, on the far right. And where is the tens place? . . . Yes, two places over."** (Count and point to the tens place.) **"Let's see if we can count like Super 10 bear."**

Demonstrate/Model

"Look at the first box. How many bundles of ten are there?" (Count and point to each bundle.) **"There are four bundles of ten. Because there are four tens, I'm going to write a four in the tens column. How many ones are there? There are two ones. I'll write two in the ones column."**

Guided Practice

"You do the next box. Count the bundles of tens and write the number in the ones column. I'll watch and help."

Independent Practice

"You've got the hang of it now. So, go ahead and finish this page and worksheet four on your own." (Distribute worksheet four.)

Criterion

On part one, 12 our of 14 or 86% performed correctly. On part two, 11 out of 13 or 85% performed correctly. Repeat practice until criterion is met on both parts of the lesson.

Administer 1-minute probe. Chart performance.

Appendix D

Sample Teaching Script for Abstract Lesson

Lesson Eight: Identifying Tens and Ones Abstractly With Worksheet and Cards

Materials: place value cards, vis-a-vis pen, worksheet, probe, chart

Advance Organizer

"You've done so well with counting tens and ones using pictures that you're now ready to do place value without pictures. After today's lesson you'll understand tens and ones without pictures. That's great. After we practice place value with numbers only, we'll do our 1-minute timing."

Demonstrate/Model

(Hold up place value card showing 56.) **"This number is 56. In the number 56, the number two places over stands for tens. So there are five tens in this number. The number on the far right stands for ones. So there are six ones in this number. I'll write the tens in the ten column and the ones in the one column on the opposite side of the card."** (After writing in the numbers, give the student a stack of place value cards.)

Guided Practice

"You look at the number and decide how many tens and ones there are in the number. Then turn your card over and write your answer in the correct columns. I'll watch and help as needed . . . Good job."

Independent Practice

"Now complete the stack by yourself. When you've finished practicing with the cards, I'd like you to complete this worksheet." (Distribute worksheet.) **"Do your best work on both the cards and the worksheet."**

Criterion

On the place value cards, the student must get 16 out of 20 or 80% performed correctly. On the worksheet the student must get 12 out of 15 or 80% performed correctly.

Administer 1-minute probe. Chart performance.